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CHINA UNIVERSITY OF GEOSCIENCES

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主办单位: 湖北省物理学会
承办单位: 中国地质大学 (武汉)
2023年10月27日-29日 湖北·武汉

有关一个简单旋转偶极子的电磁场角动量密度的理论分析

Angular momenta of a rotational dipole and more

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NSFC 11804087, 41974195; 2023CFB553, Q20201006

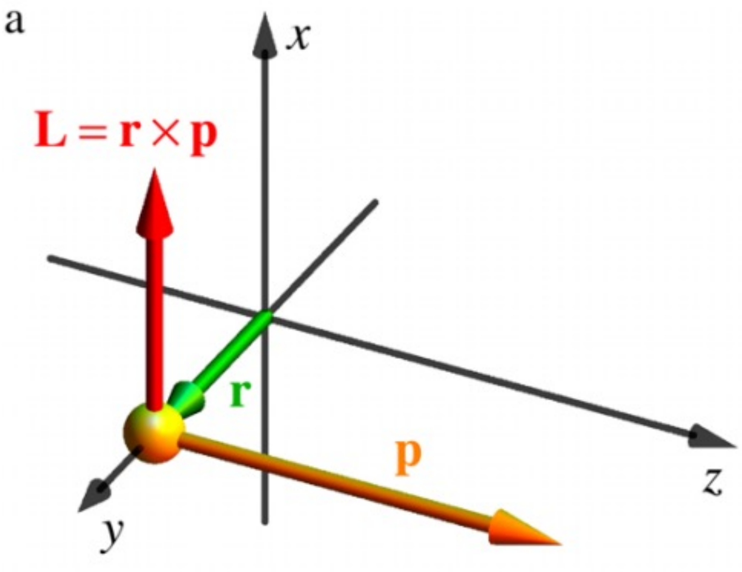
Ack: S. Horsley(Exeter), B. Vial(Imperial), Xu D.-H.(重庆大学), Xu H.-Y. (兰州大学), Xu L. (安徽大学), Du L. (深圳大学).

Optics Group in Hubei Univ　湖北大学光学课题组

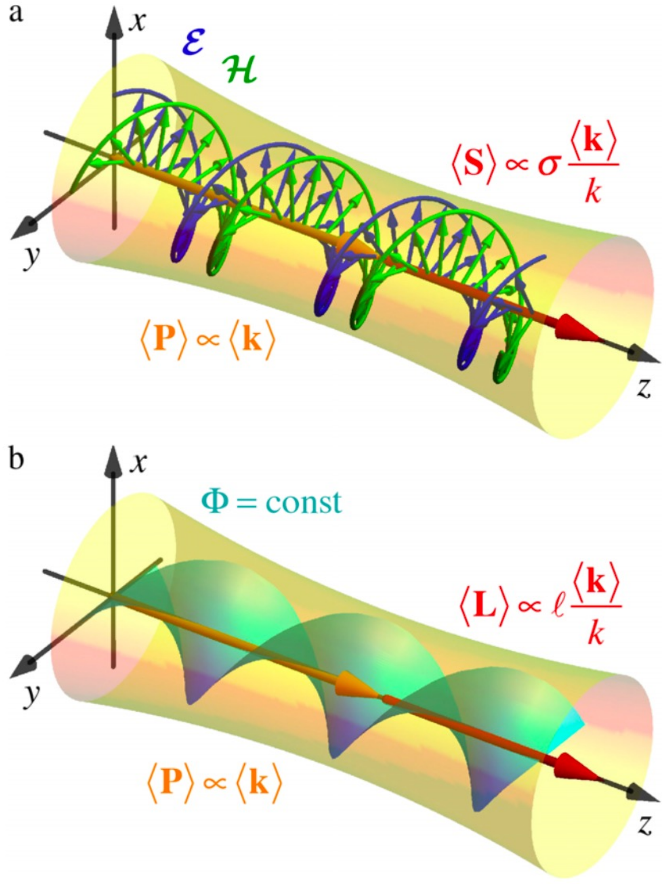
Contents

1.Angular momenta for light: orbital and spin parts

光的角动量：轨道与自旋



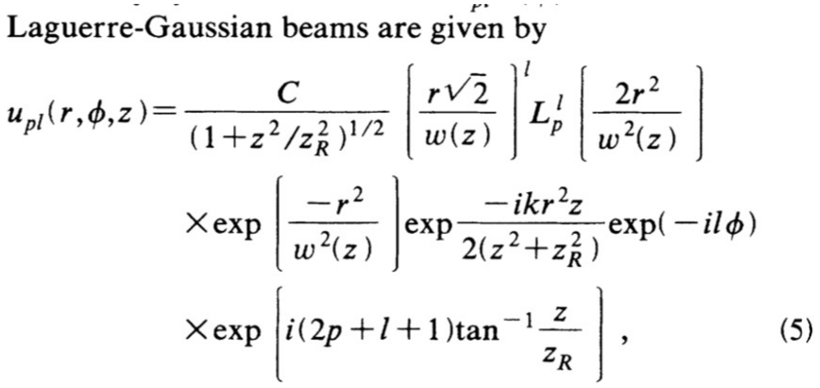
1909 Poynting：analogue



SAM in light beam

$$\vec{S} = \frac{1}{4\omega}\Im[\epsilon \vec{E}^* \times \vec{E} + \mu \vec{H}^* \times \vec{H}]$$

OAM in light beam



Laguerre-Gaussian beams are given by

$$u_{pl}(r,\phi,z)=\frac{C}{(1+z^2/z_R^2)^{1/2}}\left[\frac{r\sqrt{2}}{w(z)}\right]^l L_p^l\left[\frac{2r^2}{w^2(z)}\right] \times\exp\left[\frac{-r^2}{w^2(z)}\right]\exp\frac{-ikr^2z}{2(z^2+z_R^2)}\exp(-il\phi) \times\exp\left[i(2p+l+1)\tan^{-1}\frac{z}{z_R}\right], \qquad (5)$$

Bliokh K Y, Nori F. Physics Reports, 2015, 592: 1-38.
Allen L, *et al*, Phys. Rev. A, 1992, 45(11): 8185.

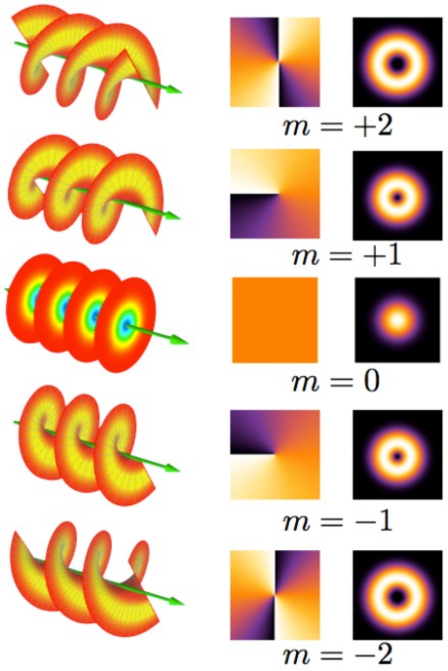
2. Optical AM vs. mechanic antenna

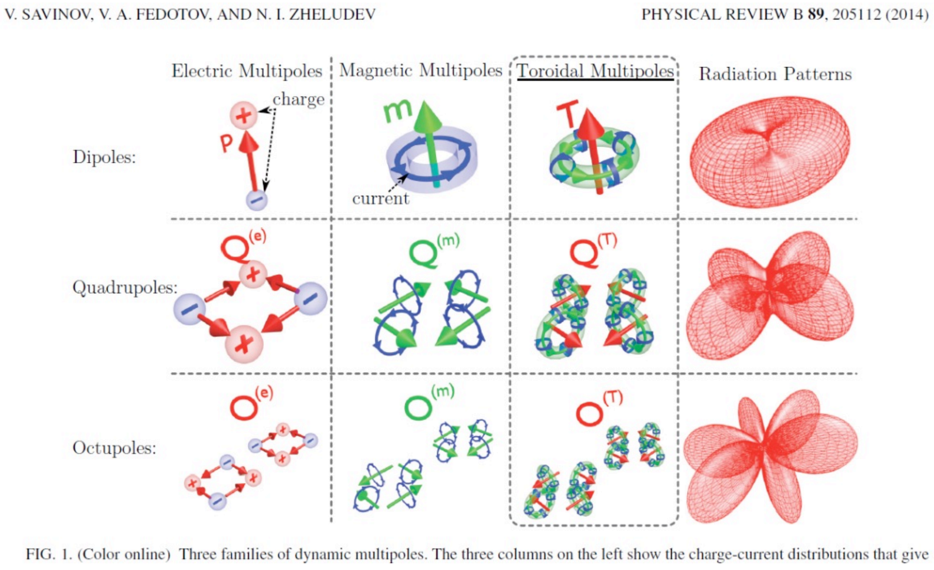
涡旋光场 与 机械天线

Vortices

multipoles

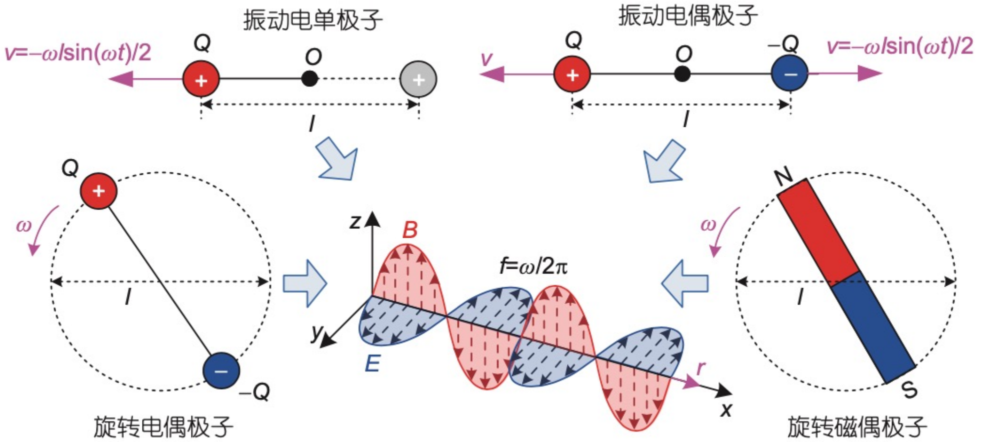
Mechanic antenna




V. SAVINOV, V. A. FEDOTOV, AND N. I. ZHELUDEV

PHYSICAL REVIEW B 89, 205112 (2014)

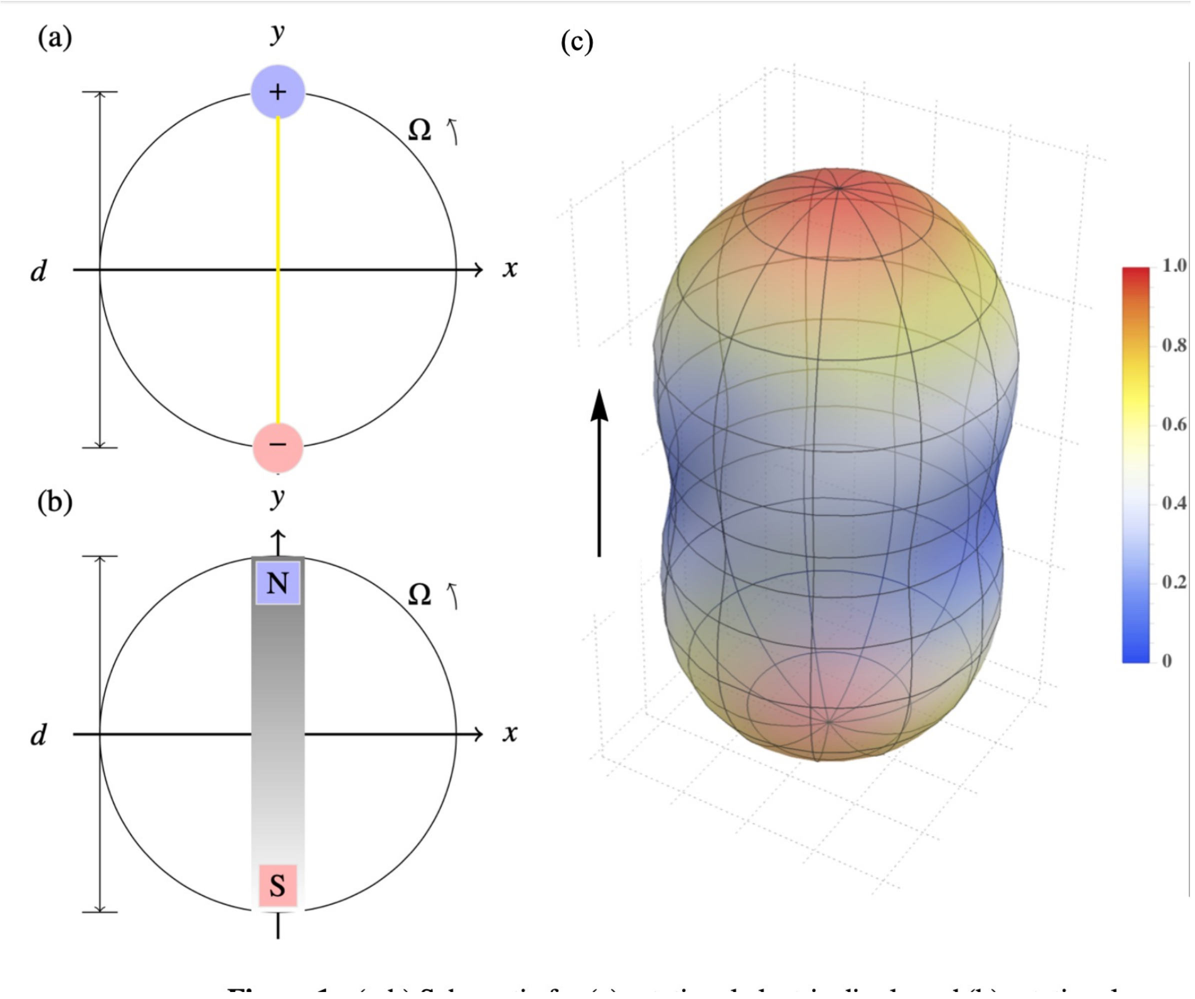
FIG. 1. (Color online) Three families of dynamic multipoles. The three columns on the left show the charge-current distributions that give



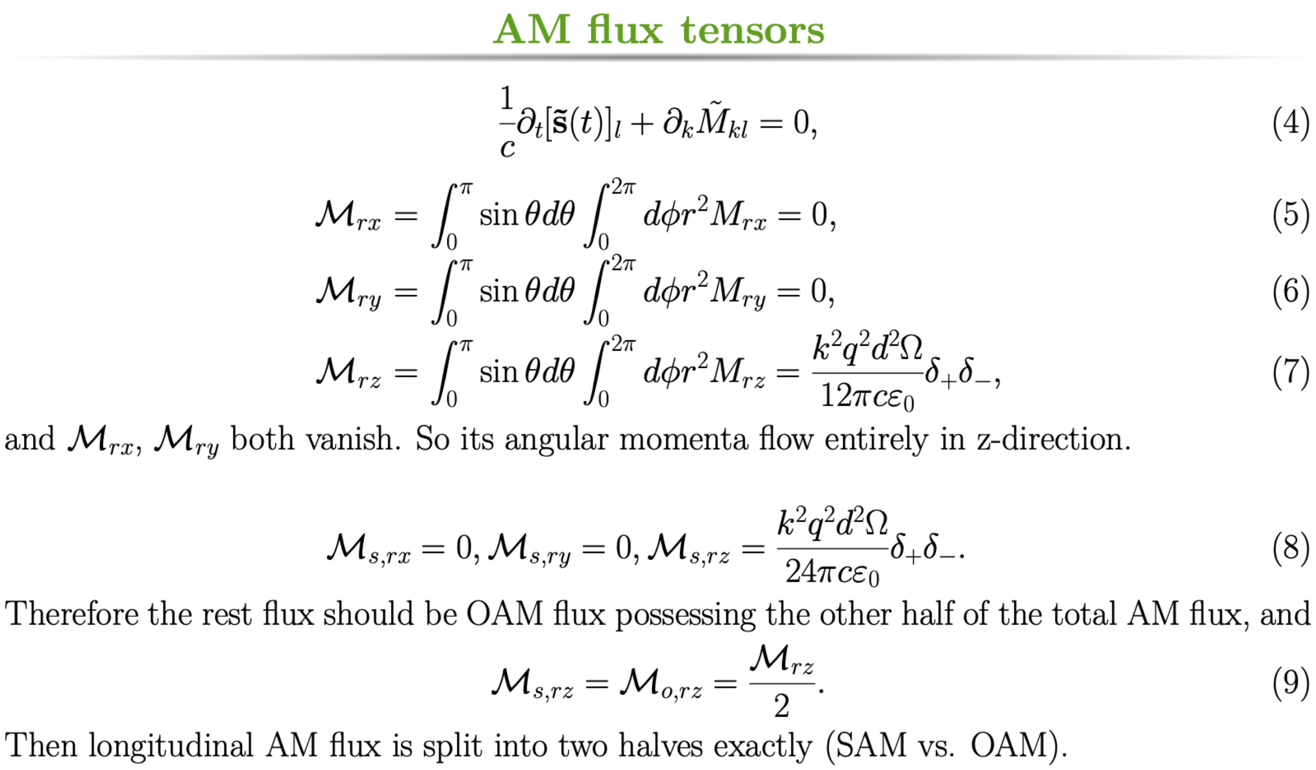
https://en.wikipedia.org/wiki/Optical_vortex

周强，施富强*，等，中国科学, 50, p69 (2020)

3. Optical angular momenta: democratic OAM and SAM



$$\mathbf{p} = \frac{qd}{2}[(-i\hat{x} + \hat{y})\delta(\omega - \Omega) + (i\hat{x} + \hat{y})\delta(\omega + \Omega)],$$



AM flux tensors

$$\frac{1}{c}\partial_t[\tilde{\mathbf{s}}(t)]_l + \partial_k \tilde{M}_{kl} = 0, \tag{4}$$

$$\mathcal{M}_{rx} = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi r^2 M_{rx} = 0, \tag{5}$$

$$\mathcal{M}_{ry} = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi r^2 M_{ry} = 0, \tag{6}$$

$$\mathcal{M}_{rz} = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi r^2 M_{rz} = \frac{k^2 q^2 d^2 \Omega}{12\pi c \varepsilon_0}\delta_+\delta_-, \tag{7}$$

and $\mathcal{M}_{rx}$, $\mathcal{M}_{ry}$ both vanish. So its angular momenta flow entirely in z-direction.

$$\mathcal{M}_{s,rx} = 0, \mathcal{M}_{s,ry} = 0, \mathcal{M}_{s,rz} = \frac{k^2 q^2 d^2 \Omega}{24\pi c \varepsilon_0}\delta_+\delta_-. \tag{8}$$

Therefore the rest flux should be OAM flux possessing the other half of the total AM flux, and

$$\mathcal{M}_{s,rz} = \mathcal{M}_{o,rz} = \frac{\mathcal{M}_{rz}}{2}. \tag{9}$$

Then longitudinal AM flux is split into two halves exactly (SAM vs. OAM).

Y. Mao, Y. Liu* and H. Lin*, IOP **J. Phys. Commun**. 5,125012 (2021).

More on OAM and SAM

Electric field plot: OAM

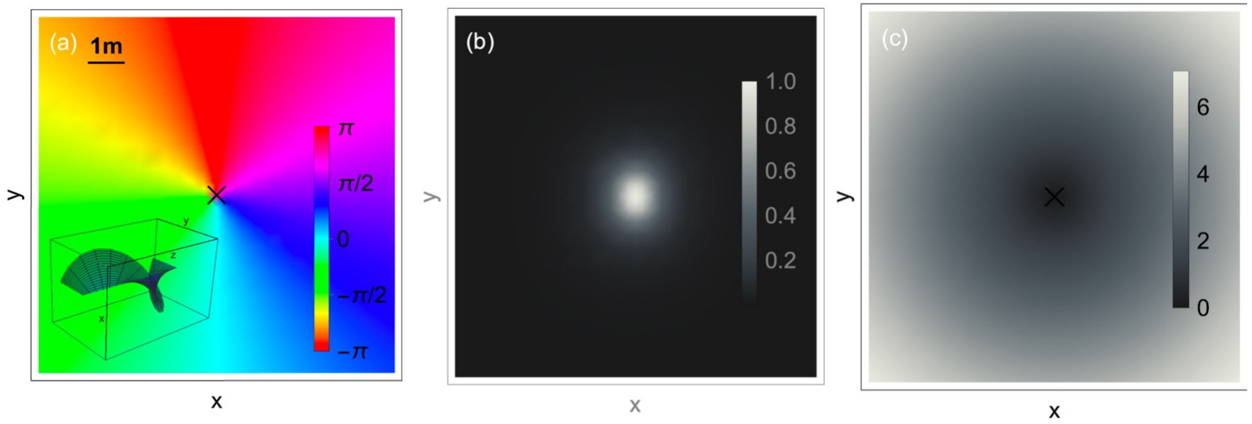


Figure: Far fields (a) Phase plots for E_z on plane $z = z_0$. Inset of (a): helical phase of order 1 for far-region electric field. (b) $|\mathbf{E}(x, y)|_{z=z_0}$. (c) $E_z(x, y)|_{z=z_0}$.

Polarization states: SAM

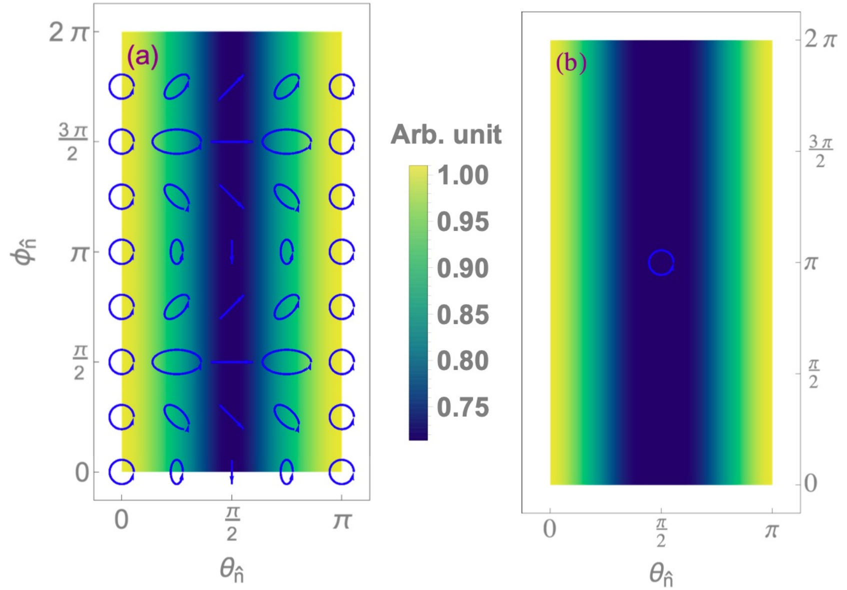
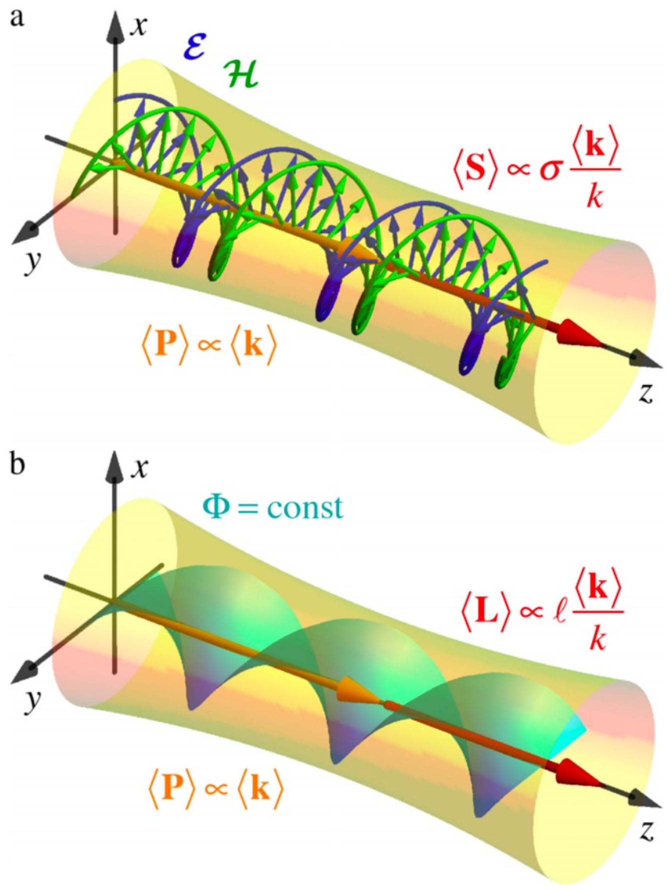


Figure: $|\mathbf{E}|(\theta_{\hat{n}}, \phi_{\hat{n}})$ and $|\mathbf{H}|(\theta_{\hat{n}}, \phi_{\hat{n}})$ superimposed with polarization circles: (a) electric fields and (b) magnetic fields. (b) All polarizations for magnetic fields are uniformly left-circular.

Y. Mao, Y. Liu* and H. Lin*, IOP **J. Phys. Commun**. 5,125012 (2021).

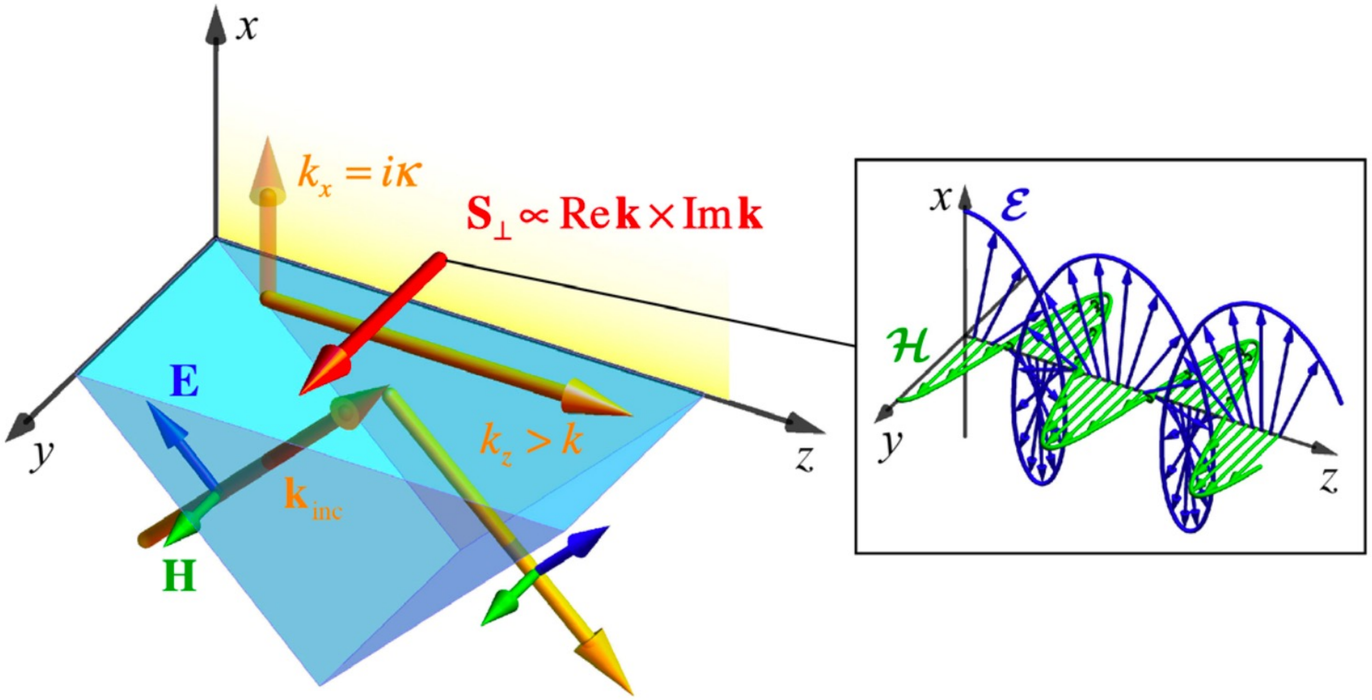
recap1.Angular momenta for light: orbital and spin parts

光的角动量：轨道与自旋



SAM in light beam

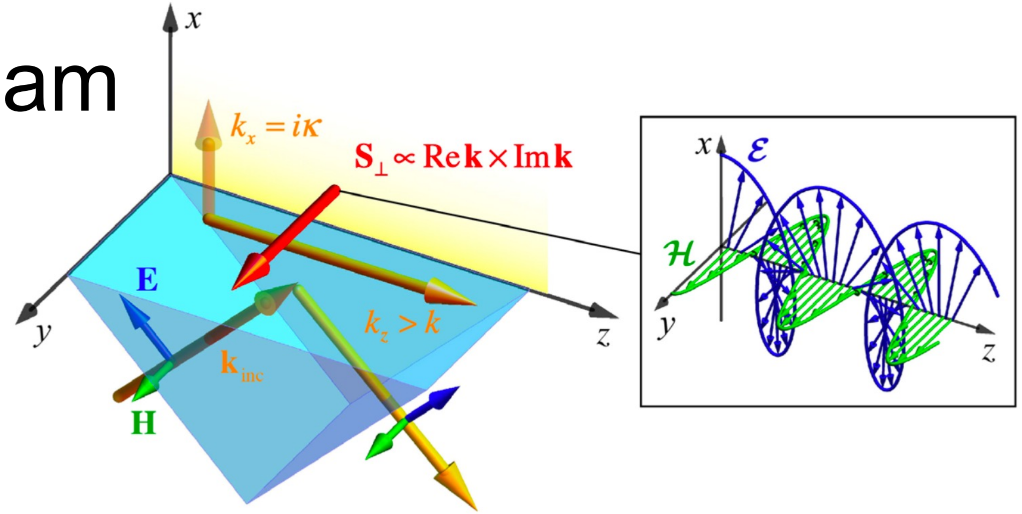
$$\vec{S} = \frac{1}{4\omega}\Im[\epsilon \vec{E}^* \times \vec{E} + \mu \vec{H}^* \times \vec{H}]$$

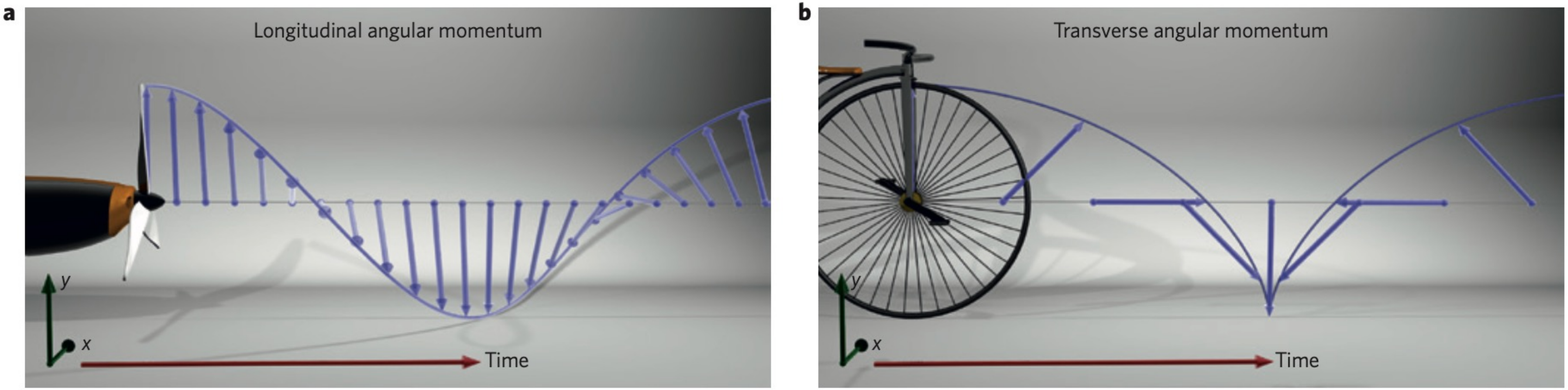


Transverse SAM in light beam

Bliokh K Y, Nori F. Physics Reports, 2015, 592: 1-38.
Allen L, *et al*, Phys. Rev. A, 1992, 45(11): 8185.

4. Transverse SAM in light beam





Andrea Aiello, et al., Nature Photonics **9**, 789 (2015).

4. Expansion on spin-momenta locking:
more than from curl of momentum

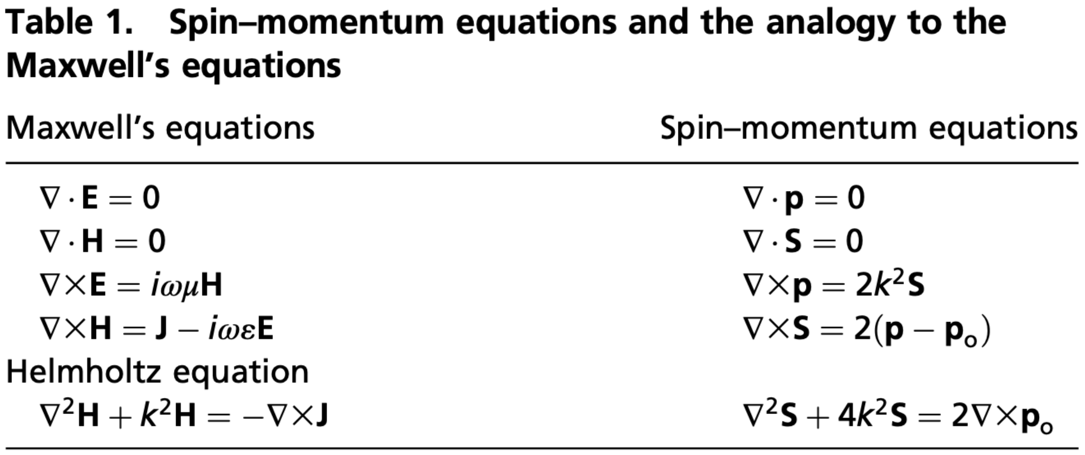
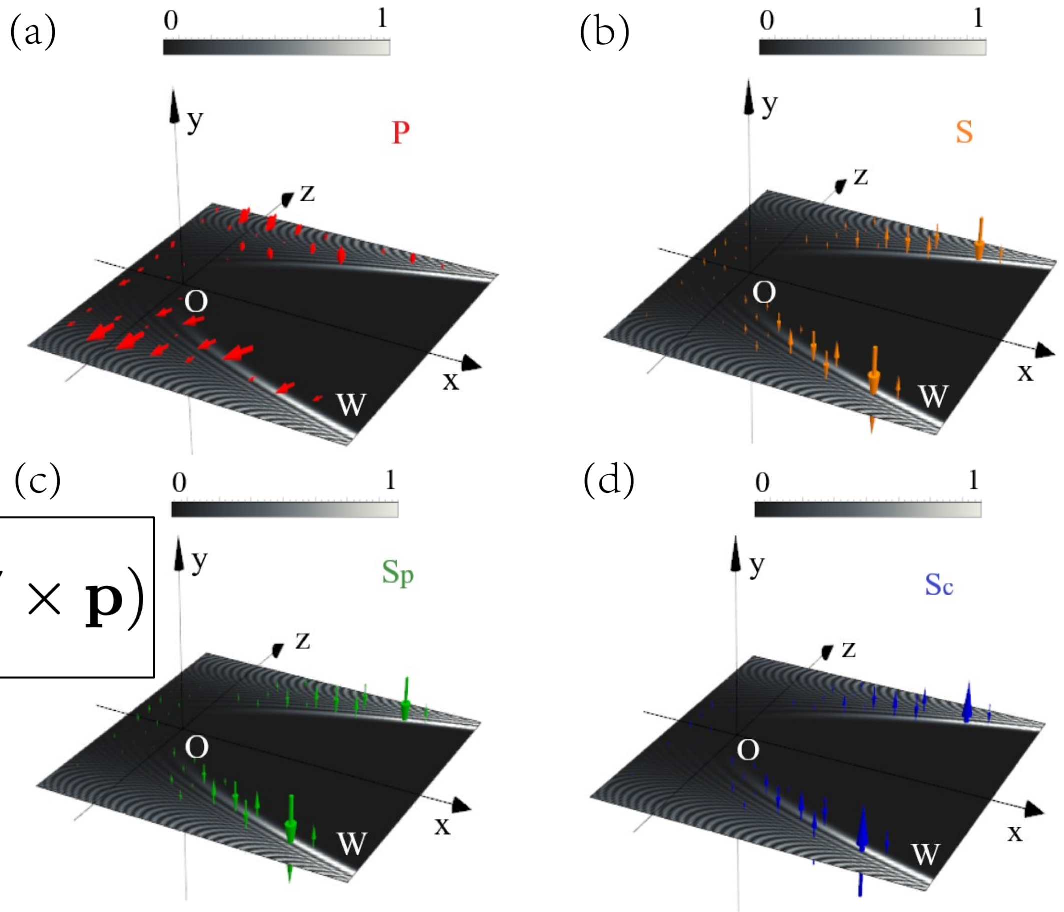


Table 1. Spin–momentum equations and the analogy to the Maxwell's equations

Maxwell's equations	Spin–momentum equations
$\nabla\cdot\mathbf{E}=0$	$\nabla\cdot\mathbf{p}=0$
$\nabla\cdot\mathbf{H}=0$	$\nabla\cdot\mathbf{S}=0$
$\nabla\times\mathbf{E}=i\omega\mu\mathbf{H}$	$\nabla\times\mathbf{p}=2k^2\mathbf{S}$
$\nabla\times\mathbf{H}=\mathbf{J}-i\omega\varepsilon\mathbf{E}$	$\nabla\times\mathbf{S}=2(\mathbf{p}-\mathbf{p}_o)$
Helmholtz equation	
$\nabla^2\mathbf{H}+k^2\mathbf{H}=-\nabla\times\mathbf{J}$	$\nabla^2\mathbf{S}+4k^2\mathbf{S}=2\nabla\times\mathbf{p}_o$

P. Shi, L. Du*, et al., **PNAS**, 10.1073/pnas.2018816118 (2021).

$$\boxed{\mathbf{s}_p = \frac{1}{k^2}(\nabla\times\mathbf{p})}$$



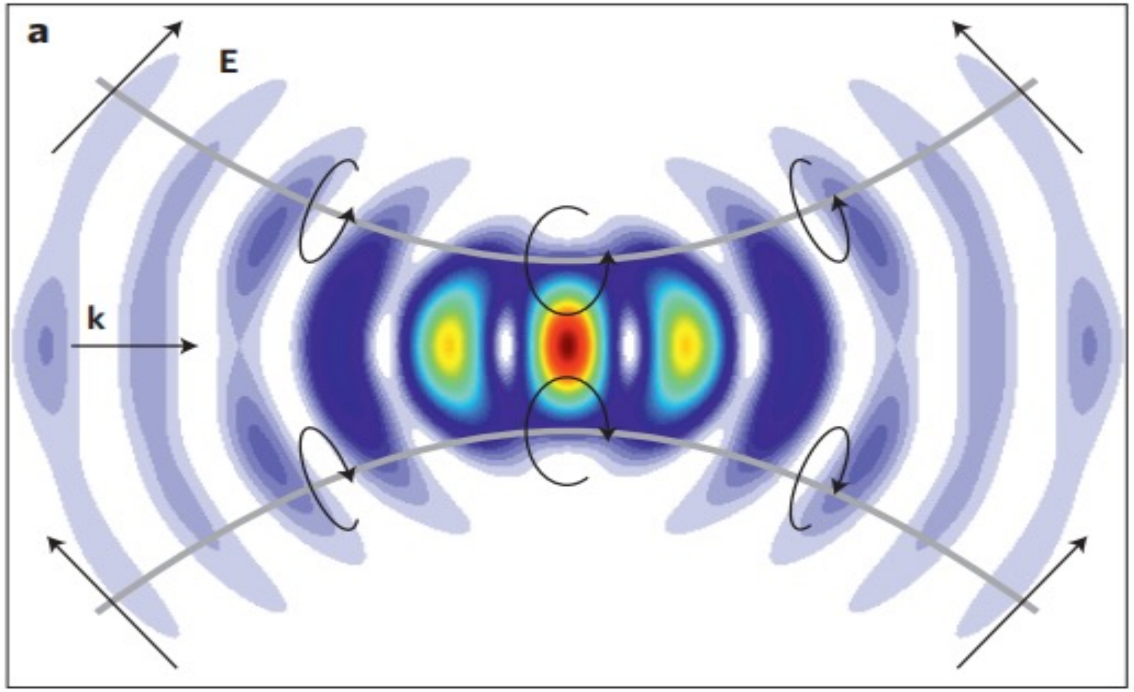
Evanescent wave in guided structures

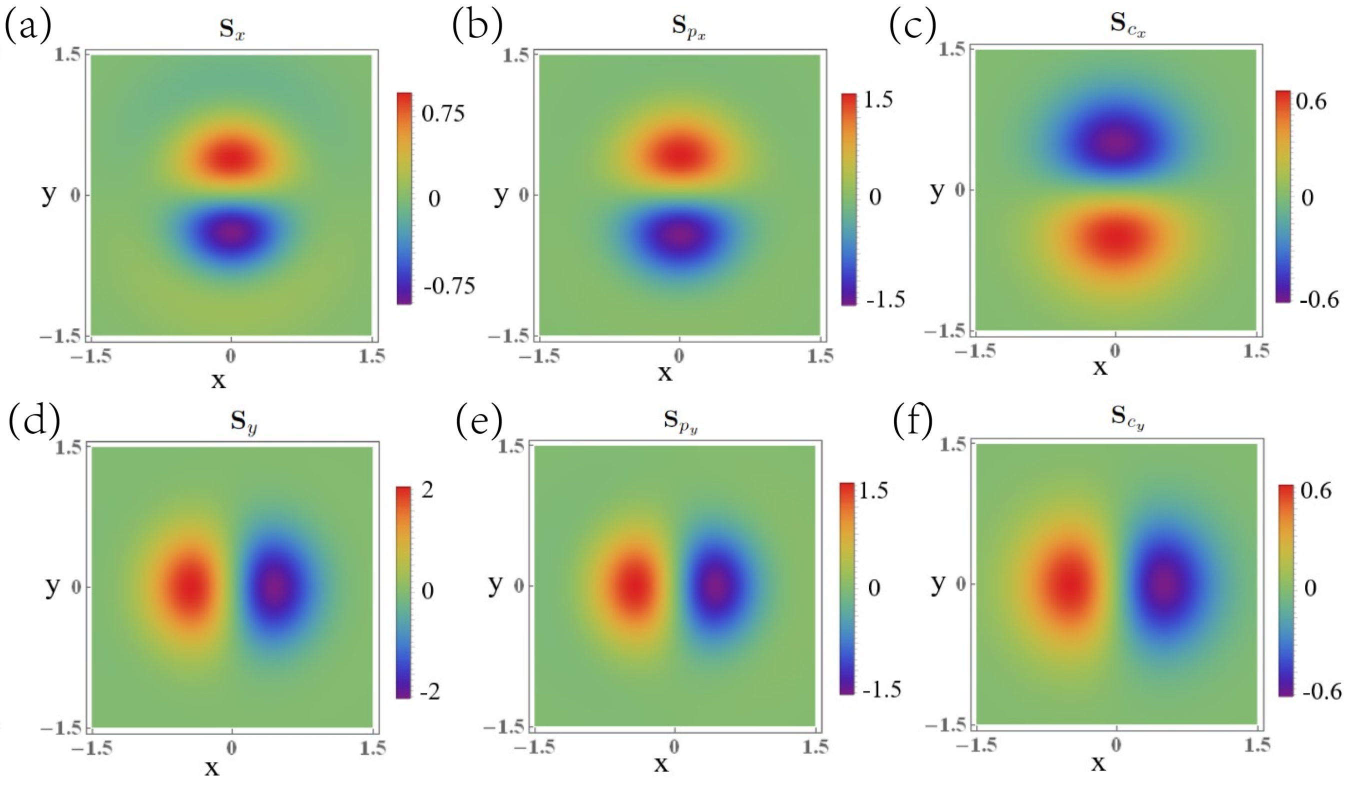
Z.-K. Xiong, Wang Z.-L., Y. Liu*, et al., arxiv:2310.06664

For All waves including propagating waves in free space:
Curl of momentum DO NOT give the full transverse SAM.

Expansion on spin-momenta locking:

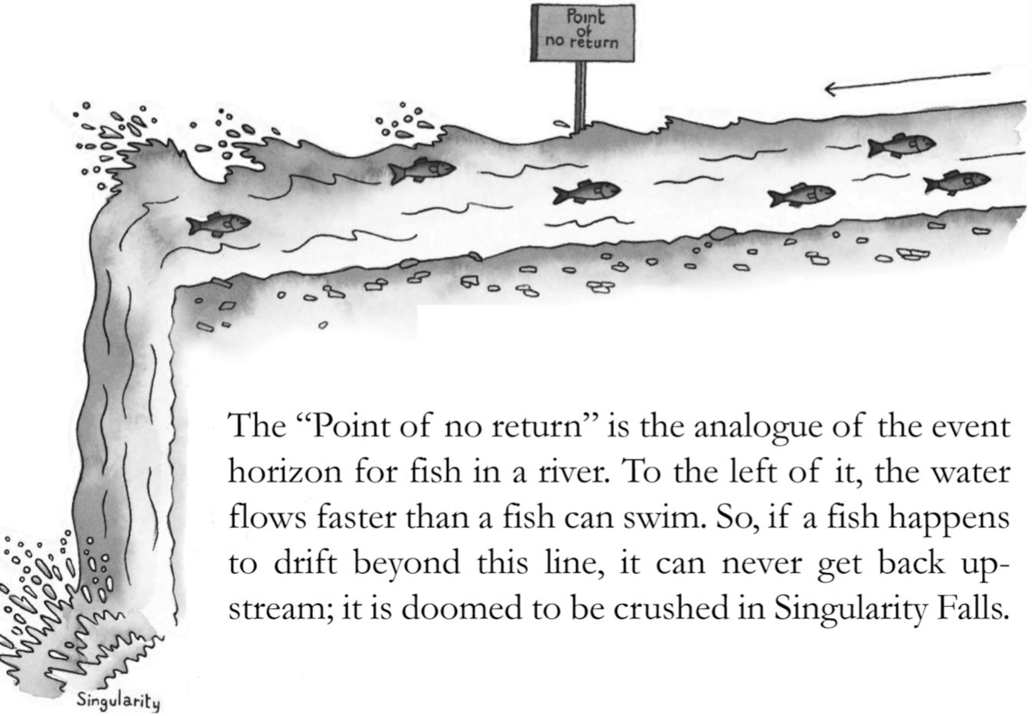
more than from curl of momentum





Z.-K. Xiong, Wang Z.-L., Y. Liu*, et al., arxiv:2310.06664

A bit more…



The "Point of no return" is the analogue of the event horizon for fish in a river. To the left of it, the water flows faster than a fish can swim. So, if a fish happens to drift beyond this line, it can never get back upstream; it is doomed to be crushed in Singularity Falls.

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$\nabla \times \vec{E} =$

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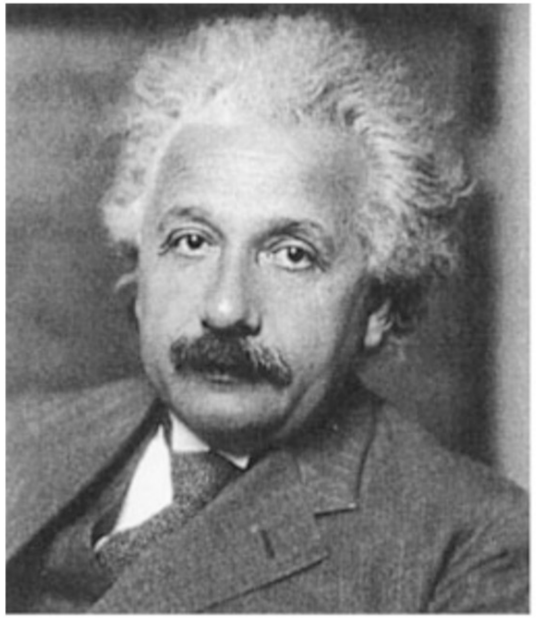
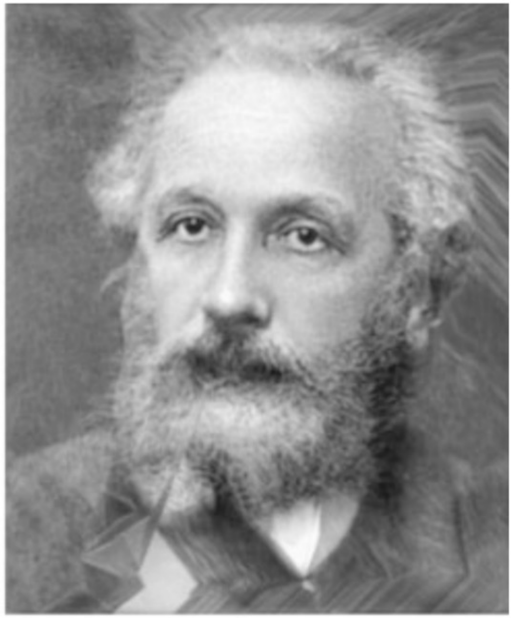
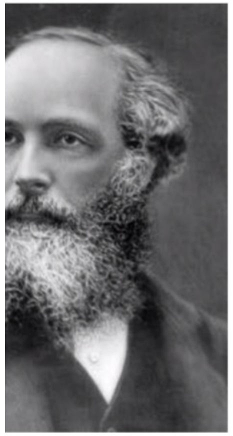
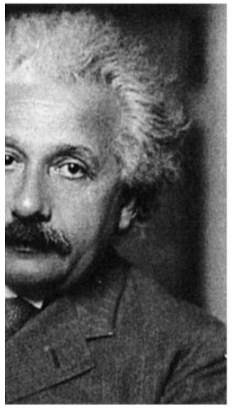
$\vec{D} =$





lativity





--U. Leonhardt, Essential Quantum Optics, Cambridge Press
--U. Leonhardt, Ch10, in Analogue Gravity Phenomenology, Springer Press, 2013. -

4. Conclusion

(1) a rotational dipole and its radiation features on AM

(2) Expansion on spin-momentum relation

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Thank you for your time.

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